

Appendix to:

Why Do Demand Curves for Stocks Slope Down?

Empirical Tests

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1 Empirical Tests

The most unique implications of our model are perhaps Implication 2 and Implication 3, since they link the fee of the intermediary to the slope of the demand curve. As one possible way to test these implications, we start with the observation that small-cap money managers generally charge higher fees than large-cap managers, presumably due to higher information costs for small firms. Hence, small-cap stocks should have steeper demand curves than large-cap stocks. We perform this test by comparing the demand curves implied by index changes to the Russell 2000, which is a small-cap index, and the S&P 500, which is a large-cap index.

A straightforward cross-sectional test is provided by Implication 1, which links the idiosyncratic risk of a stock to the slope of its demand curve. We test this prediction both for Russell 2000 stocks and for S&P 500 stocks. The Russell 2000 provides a much larger sample of event stocks than the S&P 500 with hundreds as opposed to tens of stocks each year. The Russell 2000 data are also relatively untouched by academic researchers, as almost all U.S. index studies have focused on the S&P 500.

We start with the cross-sectional tests for the two indices because as a byproduct they also yield the slopes of the demand curves.

1.1 The Russell 2000

1.1.1 Background

Each year, Frank Russell Co. sorts all publicly listed U.S. firms based on the market capitalizations of their public floats after the close of trading on May 31. The largest 1,000 stocks form the Russell 1000 index and the largest 3,000 stocks form the Russell 3000 index. The Russell 2000 index consists of all Russell 3000 stocks minus all Russell 1000 stocks. The Russell 2000 thus measures the performance of small-cap stocks, and for that purpose it is still the most commonly quoted index. The new index composition becomes effective on July 1, and that will generally not change for the next 12 months unless the stock is delisted from its exchange. The index weight of a stock is determined by the market value of its public float as defined by Russell.

The index funds mechanically tracking the Russell 2000 generally update their portfolios on the last trading day of June in order to minimize their tracking error. The index

composition is announced much earlier in June though, and any active market participant can infer the index composition based on market prices on May 31. Since the index selection rule is entirely mechanical, it is in fact possible for market participants to have a good idea of the future index composition even before May 31. However, historically most of the price impact associated with index changes has taken place in June, so in order to minimize the effect of noise and also not to create any forward-looking bias, we focus on abnormal returns in June.

Stocks can enter or leave the Russell 2000 from below or above, i.e. if they cross either the lower cutoff (around \$130 million in 2002) or the upper cutoff (around \$1.3 billion in 2002). Stocks crossing the upper cutoff experience a demand shock due to the different fractions held by Russell 1000 and Russell 2000 indexers. We focus on stocks added or deleted from below because it represents a cleaner experiment.

Our data for the Russell indices is obtained from Frank Russell Co. The firm has followed its current annual reconstitution methodology starting in 1990, so we use data from 1990 to 2002.

1.1.2 Methodology

We want to test whether the idiosyncratic risk of a stock affects the price impact around index addition. We choose our event window as June 1 to June 30. Hence, we test if the abnormal return in June on a stock added to (or deleted from) the index from below is positively related to the idiosyncratic risk of the stock.

To estimate idiosyncratic risk, we use 6 months of daily data from CRSP from November 1 to April 30. We require a minimum of 2 months of valid return observations. We regress the stock's daily excess return on the three factors of Fama and French.¹ We define idiosyncratic risk as the root mean squared error of this regression.² We also take the market equity of every firm on April 30 in order to obtain a value that is not affected by the anticipation of the index event.

Since it is possible that the level of idiosyncratic risk is also related to the cross-correlations of stocks, e.g. stocks with high idiosyncratic risk tend to move together, we need to control for this comovement of stocks with similar idiosyncratic risk. The market

¹http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

²We also ran all tests with the market model and obtained very similar results.

equity of a firm can also plausibly be associated with the slope of the demand curve, so we control for that as well.

Hence, we form a 10×5 matrix of control portfolios based on market equity and idiosyncratic risk. We pick all stocks in CRSP representing ordinary common shares of U.S. firms on April 30, and we sort them into 10 deciles based on the Fama-French breakpoints for market equity that month. Having estimated the idiosyncratic risk of each stock as described before, we then subdivide each market equity decile into quintiles based on idiosyncratic risk. The procedure is similar to the one used by Fama and French (1992) for market equity and beta. We perform a sequential sort rather than an independent sort because idiosyncratic risk and market equity have a high negative correlation (about -0.5 in a typical cross-section for idiosyncratic risk and log of market equity), so an independent sort would tend to cluster the stocks in the cells around the diagonal. After all, our purpose is to distinguish between levels of idiosyncratic risk within each size decile, and except for the bottom and top size deciles the correlation between the two within a size decile is relatively small (generally between -0.1 and 0). For a similar reason, we use all stocks (and not just NYSE stocks) for idiosyncratic risk breakpoints, since a large fraction of our event stocks are not NYSE stocks. Some sample statistics for the control portfolios are in the appendix.

We then compute the return on each control portfolio for each trading day in June. On May 31, we set the portfolio weights based on market capitalization on April 30. We calculate the buy-and-hold return on this portfolio for each day in June, assuming that when a stock no longer has valid CRSP return data we reinvest that wealth in the remaining portfolio.

Having determined the breakpoints for market equity and idiosyncratic risk, we can assign each event stock to its corresponding cell in the 10×5 matrix. We then define the cumulative abnormal return on a stock in June as the difference between the cumulative stock return and the corresponding cumulative control portfolio return.

Since all index changes each year occur at the same time, the abnormal returns are likely to exhibit significant cross-correlation. To get around this issue, we run a Fama-MacBeth regression for the 14 annual cross-sections of data covering the years 1990-2002. In other words, using all stocks within a year, we regress the abnormal stock return on idiosyncratic risk and log of market equity. Then we compute the time-series means of these regression

coefficients and the standard errors for the means.

We use both additions and deletions for the analysis. Since the index-induced demand shock for deletions has a negative sign, we simply multiply the CARs for deletions by -1 and then lump them together with the additions. This allows us to use a relatively large sample in our regressions.

1.1.3 Results

For idiosyncratic risk, the univariate Fama-MacBeth regression produces a coefficient of 0.041 and a significant t -statistic of 3.53 (Table 7). For market equity, the univariate regression produces a coefficient of -0.021 with a t -statistic of -1.31 . The bivariate regression reduces the magnitude of each estimate due to the negative correlation between the two explanatory variables. We then get a coefficient of 0.037 for idiosyncratic risk and -0.012 for market equity, with t -statistics of 2.48 and -0.72 , respectively.

Economically the coefficient of idiosyncratic risk implies that an increase of 10% in annual idiosyncratic volatility would increase the price impact of Russell 2000 addition by 0.4%, or about 10% of the average price impact over this period. Given that the index premium at the end of the sample is about twice the sample average, this could translate to an almost 1% difference in CAR due to a 10% difference in idiosyncratic volatility. This is not a trivial magnitude, especially as our coefficient estimate is likely to be biased down due to the noisy measurement of idiosyncratic risk.

1.2 The S&P 500

1.2.1 Background

Unlike the Russell indices, the S&P 500 is updated at apparently random times throughout the year, with no significant concentrations around any particular days. The criteria for index membership include market capitalization, liquidity, size of the public float, and industry representation, but in the end both the firms involved and the exact timing of the changes are decided behind closed doors and somewhat subjectively by the S&P index membership committee.

Index changes are typically announced five trading days before the effective day. Most of the associated price jump occurs immediately after the announcement, but surprisingly a nontrivial part of the price effect takes place gradually during the intervening days between

the announcement and effective day. Prior to October 1989, the index changes became effective immediately after the announcement. In order not to mix these potentially different regimes, we restrict ourselves to data from January 1990 to December 2000.

In fact, a similar test, albeit with a very different implementation, has already been carried out with S&P 500 data for the 1976-1989 period by Wurgler and Zhuravskaya (2002). The authors find evidence of a link between idiosyncratic risk and the price impact around index addition. We want to verify this result with more recent data, especially as the fraction of mechanical indexers has grown so much since the 1976-1989 period, and with our empirical methodology. This also allows us to link the S&P 500 results to the Russell 2000 results.

1.2.2 Methodology

As before, we want to test whether idiosyncratic risk is related to the price impact around index addition. For the event window, we choose the five trading days leading up to the official index change. We do not have the actual announcement dates of each addition, but our approximation does match the event dates for a typical index addition, and elsewhere it will work against us.

We try to follow our procedure for the Russell 2000 as closely as possible. We take the last end-of-month market value of equity at least one month before the effective day of the index change. We choose the 6-month period ending with the measurement day for market equity, and we estimate idiosyncratic risk in this period as the root mean squared error of a regression of daily excess returns on the three factors of Fama and French. All additions in the sample have at least 2 months of return observations in the estimation period.

We use the same 10×5 matrix of control portfolios as before. Naturally the S&P 500 additions end up in the large-cap cells in the matrix. The cumulative abnormal return on a stock in the event window is then the difference between its own cumulative return and the cumulative return of its benchmark portfolio (matched on market equity and idiosyncratic risk).

For the S&P 500 the event windows are more or less randomly distributed throughout the year, so cross-correlations of abnormal returns are not likely to be an important issue here. Hence, each observation represents an independent data point and we can regress all the observations on the explanatory variables in one cross-section. We perform such

a regression, using White's heteroskedasticity-consistent standard errors. We also plug in a dummy variable for each year to account for the increasing time trend in the index premium.

1.2.3 Results

The regression results are shown in Table 10. Idiosyncratic risk turns out to be statistically significant both in the univariate regression ($t = 4.50$) and in the bivariate regression ($t = 3.47$). Market equity also turns out to be statistically significant ($t = -3.42$ and -2.06). As before, the bivariate regression shrinks each coefficient estimate and its t -statistic toward zero due to the negative correlation between the explanatory variables. When the year dummies are not present, the coefficient of market equity is somewhat greater as the increasing size of event stocks picks up some of the increasing time trend in the CARs over the years.

The coefficient of idiosyncratic risk is about 0.2, meaning that an increase in annualized idiosyncratic volatility of 10% would increase the abnormal return around index changes by 2 percentage points. This certainly has economic significance, especially as the estimates are still likely to be biased down due to measurement error. For market equity, the coefficient of -0.044 tells us that increasing the market capitalization of a stock by a factor of 10 reduces its expected price impact around an index change by 4.4 percentage points.

1.3 Elasticity of Demand

We can estimate the price elasticity of demand for a stock using the most recent numbers from the year 2000. For the S&P 500, we find $\frac{\Delta Q}{\Delta P} = -0.7$. This is relatively close to the unit price elasticity of demand estimated by Shleifer (1986). The most significant source of noise in the estimate comes from the share of indexers: while it is relatively easy to estimate the share of purely mechanical indexers, a very large number of funds are benchmarked against the index which creates additional long-run demand for index stocks. However, it seems unlikely that any such funds would regularly rush to trade on index changes within a few days of the effective day.

For the Russell 2000, we plug in 2% for $\frac{\Delta Q}{Q}$ and 6.8% for $\frac{\Delta P}{P}$, which gives us an elasticity of -0.3 . The elasticity of demand therefore seems to be twice as great for the S&P 500 index changes as for the Russell 2000 index changes. The steeper demand curves for small

stocks, where the management fees are higher, are consistent with the central prediction of our model.

Of course there are also other reasons which could explain this difference, and thus we would need more data to establish the statistical significance of the management fee on the slopes of demand curves. International data from various markets would provide a natural experiment, but even then the data will be very noisy due to various other cross-country differences which also have an impact on demand curves. This lack of a large or clean sample is an unfortunate liability in any such tests which look for systematic differences across entire markets as opposed to individual assets.

2 Tables

2.1 Medians for the Benchmark Portfolios 1990-2002

Number of stocks					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small	615	615	615	615	614
2	177	177	177	177	176
3	110	110	110	110	109
4	79	79	79	79	79
5	60	60	60	60	60
6	52	52	52	52	51
7	47	47	47	47	46
8	42	42	42	42	41
9	38	38	38	38	38
Big	36	35	35	35	35

Table 1: Median number of stocks in each benchmark portfolio. The statistics are computed for all U.S. firms listed on the NYSE, AMEX, or Nasdaq, and over the period 1990-2002. The 10×5 benchmark portfolios for month t are formed first by dividing stocks into size deciles based on Fama-French breakpoints for market equity at the end of month $t - 2$. Stocks within each size decile are then sorted into quintiles based on the root mean squared error of a regression of stock returns on the three factors of Fama-French over the 6-month-period from month $t - 7$ to month $t - 2$.

Market capitalization (M)					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small	42	40	34	26	14
2	131	129	130	130	130
3	250	246	246	244	240
4	399	394	395	391	390
5	603	599	611	599	600
6	931	905	917	919	907
7	1,459	1,473	1,466	1,461	1,447
8	2,494	2,440	2,402	2,432	2,455
9	4,686	4,785	4,903	4,627	4,476
Big	22,511	24,541	19,731	20,875	17,178

Table 2: Median market capitalization of each benchmark portfolio. The statistics are computed for all U.S. firms listed on the NYSE, AMEX, or Nasdaq, and over the period 1990-2002. The 10×5 benchmark portfolios for month t are formed first by dividing stocks into size deciles based on Fama-French breakpoints for market equity at the end of month $t - 2$. Stocks within each size decile are then sorted into quintiles based on the root mean squared error of a regression of stock returns on the three factors of Fama-French over the 6-month-period from month $t - 7$ to month $t - 2$.

Idiosyncratic risk (annualized)					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small	0.32	0.55	0.75	0.99	1.61
2	0.27	0.42	0.52	0.64	0.89
3	0.24	0.36	0.45	0.56	0.78
4	0.22	0.31	0.39	0.49	0.69
5	0.20	0.29	0.36	0.46	0.65
6	0.18	0.26	0.32	0.42	0.60
7	0.16	0.23	0.29	0.37	0.54
8	0.16	0.22	0.27	0.34	0.50
9	0.15	0.20	0.24	0.30	0.44
Big	0.15	0.19	0.22	0.26	0.35

Table 3: Median idiosyncratic risk for each benchmark portfolio. Idiosyncratic risk is defined as the root mean squared error of a regression of stock returns on the three factors of Fama-French. The statistics are computed for all U.S. firms listed on the NYSE, AMEX, or Nasdaq, and over the period 1990-2002. The 10×5 benchmark portfolios for month t are formed first by dividing stocks into size deciles based on Fama-French breakpoints for market equity at the end of month $t - 2$. Stocks within each size decile are then sorted into quintiles based on idiosyncratic risk computed over the 6-month-period from month $t - 7$ to month $t - 2$.

Share of stocks with zero trading volume					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small	0.194	0.108	0.066	0.058	0.098
2	0.070	0.047	0.033	0.020	0.016
3	0.028	0.022	0.016	0.008	0.007
4	0.012	0.008	0.007	0.007	0.004
5	0.004	0.003	0.002	0.001	0.001
6	0.003	0.001	0.001	0.000	0.000
7	0.002	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000
Big	0.000	0.000	0.000	0.000	0.000

Table 4: Median share of stock-days with zero trading volume in each benchmark portfolio. The statistics are computed for all U.S. firms listed on the NYSE, AMEX, or Nasdaq, and over the period 1990-2002. The 10×5 benchmark portfolios for month t are formed first by dividing stocks into size deciles based on Fama-French breakpoints for market equity at the end of month $t - 2$. Stocks within each size decile are then sorted into quintiles based on the root mean squared error of a regression of stock returns on the three factors of Fama-French over the 6-month-period from month $t - 7$ to month $t - 2$.

2.2 Russell 2000

Number of stocks					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small	36.2	49.6	48.8	52.8	33.6
2	43.5	45.1	53.2	56.3	56.0
3	12.0	13.2	20.0	25.2	33.4
4	4.1	4.3	7.0	10.8	17.9
5	1.5	2.6	2.8	7.0	10.2
6	0.5	0.4	0.6	1.6	4.9
7	0.1	0.1	0.1	0.2	1.6
8	0.1	0.2			
9					
Big					

Table 5: Russell 2000 additions in 1990-2002. This is the average number of event stocks per year for each benchmark portfolio. The event stocks are additions or deletions which have crossed the lower cutoff of the index. The benchmark portfolios are defined as before.

Year	Additions			Deletions		
	N	CAR	stderr	N	CAR	stderr
1990	283	0.026	0.012	154	-0.033	0.053
1991	377	0.023	0.010	330	-0.060	0.025
1992	408	0.021	0.011	381	-0.039	0.018
1993	319	0.016	0.010	308	-0.040	0.011
1994	400	0.004	0.011	402	-0.029	0.011
1995	322	0.041	0.012	238	-0.067	0.017
1996	364	0.028	0.012	304	-0.060	0.012
1997	386	0.057	0.010	305	-0.059	0.018
1998	370	0.040	0.011	223	0.015	0.017
1999	324	0.091	0.038	227	-0.074	0.032
2000	442	0.094	0.035	318	-0.080	0.024
2001	486	-0.008	0.021	257	-0.071	0.045
2002	352	0.066	0.025	216	-0.070	0.051

Table 6: Abnormal returns for Russell 2000 additions and deletions that cross the lower cutoff of the index. The table shows the buy-and-hold abnormal returns for all qualifying event stocks from May 31 (when the new index is determined) to June 30 (when the new index becomes effective). Stocks are given equal portfolio weights on May 31. Stocks need to have at least 2 months of CRSP return data prior to the event. The abnormal return on a stock is the difference between the stock return and the return on its control portfolio which has similar idiosyncratic risk and market equity.

Fama-MacBeth regression specification			
	1	2	3
intercept	0.013 (1.21)	0.082 (2.15)	0.035 (0.77)
sigma	0.041 (3.53)		0.037 (2.48)
$\log_{10}(ME)$		-0.021 (-1.31)	-0.012 (-0.72)

Table 7: The results of a Fama-MacBeth regression $R_{it} = \gamma_{0t} + \gamma_{1t}\sigma_{it} + \gamma_{2t}\log_{10}(ME_{it})$ for 13 cross-sections from 1990 to 2002. For each cross-section, we regress the cumulative abnormal returns on individual stocks in June on the stocks' market capitalizations and idiosyncratic volatilities estimated from November 1 to April 30. We then compute the time-series means of regression coefficients γ_{0t} , γ_{1t} , and γ_{2t} . t-statistics are reported in parentheses.

2.3 S&P 500

Number of stocks					
	Idiosyncratic risk				
Size	Low	2	3	4	High
Small					
2					
3					
4					
5			1		
6	1		1	2	
7		1	2	3	1
8	10	5	11	12	7
9	15	12	23	42	36
Big	1	1	6	13	35

Table 8: S&P 500 additions in 1990-2000. This is the number of addition event stocks for each benchmark portfolio. The benchmark portfolios are defined as before.

Year	Additions			Deletions		
	N	CAR	stderr	N	CAR	stderr
1990	11	0.002	0.024	7	-0.277	0.102
1991	10	0.129	0.024	5	-0.469	0.152
1992	6	-0.026	0.040	5	-0.409	0.090
1993	9	0.061	0.034	6	-0.053	0.046
1994	15	0.063	0.021	13	0.031	0.043
1995	23	0.079	0.018	14	-0.096	0.035
1996	21	0.094	0.024	14	-0.059	0.025
1997	24	0.110	0.018	7	-0.068	0.048
1998	37	0.135	0.017	10	-0.138	0.046
1999	39	0.105	0.022	11	-0.155	0.049
2000	50	0.151	0.029	23	-0.189	0.041

Table 9: Abnormal returns for S&P 500 additions and deletions. The table shows the buy-and-hold abnormal returns for all qualifying event stocks from 15 trading days before the announcement day to the effective day of the change. Stocks are given equal portfolio weights at the beginning. Stocks need to exist in CRSP 15 trading days before the announcement and 15 trading days after the index change. The abnormal return on a stock is the difference between the stock return and the return on the S&P 500 index.

Regression specification						
	Without year dummies			With year dummies		
	1	2	3	4	5	6
sigma	0.246		0.225	0.225		0.189
	(5.00)		(4.49)	(4.50)		(3.47)
$\log_{10}(ME)$		-0.056	-0.034		-0.068	-0.044
		(-2.77)	(-1.74)		(-3.42)	(-2.06)
df	358	358	357	348	348	347
R^2	0.102	0.033	0.113	0.133	0.105	0.148

Table 10: The results of cumulative abnormal returns for index additions and deletions in 1990-2000 regressed on the log of market equity and idiosyncratic volatility of a stock as well as year dummies. The cumulative abnormal returns are defined as the market-adjusted cumulative returns from 15 trading days before the announcement up to the effective day of the change. t -statistics based on White's heteroskedasticity-consistent standard errors are reported in parentheses.